M1 latest pyq-2 - Previous years question paper of M1 Osmania University

Mathematics- I (Osmania University)

Code No. D-2322/N/AICTE

FACULTY OF ENGINEERING

B.E. (AICTE) I – Semester(Common for All Branches) (Main & Backlog)
Examinations, March / April 2022
Subject: Mathematics - I

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each Question carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1.

(a) Discuss the convergence of the series $\sum \frac{1}{n^2}$.

(b) Obtain the fourth degree Taylor's polynomial approximation to $f(x) = e^{2x}$ about $x = 0$.

(c) Show that the following function $f(x, y)$ is continuous at the point (0,0).

$$f(x,y) = \begin{cases} \frac{2x(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(d) Evaluate the double integral $\iint_R e^{x^2} dx\, dy$, where the region R is given by $R: 2y \leq x \leq 2$ and $0 \leq y \leq 1$.

(e) Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^2$ at the point (1,2,3) in the direction of *3i+4j-5k*.

(f) If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $r = |\boldsymbol{r}|$ show that $div\left(\frac{\boldsymbol{r}}{r^3}\right) = 0$.

2. (a) Examine the convergence or divergence of the following series: $\sum \frac{x^{n+1}}{(n+1)\sqrt{n}}$.

(b) Test the convergence of the series $\sum \frac{(-1)^{n-1}}{(2n-2)!}$.

3. (a) Obtain the Taylor's polynomial approximation of degree n to the function $f(x) = e^x$ about the point $x = 0$.

(b) Using Lagrange mean value theorem, show that $1 + x < e^x < 1 + xe^x$.

4. (a) If $f(x,y) = \tan^{-1}(xy)$, find an approximate value of $f(1.1, 0.8)$ using the Taylor's series (i) linear approximation and (ii) quadratic approximation.

(b) Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

5. (a) Evaluate the integral $\iiint_R (x^2y)dx\, dy\, dz$, where the boundary R: $x^2 + y^2 \leq 1, 0 \leq z \leq 1$.

(b) Evaluate the integral $\iiint_R (2x - y - z)dx\, dy\, dz$, where the boundary R: $0 \leq x \leq 1, 0 \leq y \leq x^2, 0 \leq z \leq x + y$.

6. (a) Find the work done by the force $F = (x^2 - y^3)i + (x + y)j$ in moving a particle along the closed path C containing the curves $x + y = 0, x^2 + y^2 = 16$ and $y = x$ in the first and fourth quadrants.

   (b) Let D be the region bounded by the closed cylinder $x^2 + y^2 = 16, z = 0$ and $z = 4$. Verify the divergence theorem if $\mathbf{v} = 3x^2 i + 6y^2 j + zk$.

7. (a) Evaluate the surface integral $\iint_S F.n\, dA$ where $F = 6z\, i + 6j + 3yk$ and S is the portion of the plane $2x + 3y + 4z = 12$, which is in the first octant.

   (b) Evaluate the integral $\iint_S (\nabla \times \mathbf{v}).n\, dA$ by Stoke's theorem where $\mathbf{v} = (x^2 - y^2)i + (y^2 - x^2)j + zk$ and S is the portion of the surface $x^2 + y^2 - 2by + bx = 0, b$ constant, whose boundary lies in the x-y plane.

Code No: E-5601/N/BL/AICTE

FACULTY OF ENGINEERING

B.E. (Common to All Branches) I - Semester (AICTE) (Backlog) (New) Examination, September/ October 2023

Subject: Mathematics - I **Max. Marks: 70**

Time: 3 Hours

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each questions carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. (a) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{3n+5}{5n+4}$

(b) Find the radius of curvature of the curve $x^3 = 2y^2$ at $P(2,2)$.

(c) If $u(x,y) = \frac{y^2}{2x}$, $v(x,y) = \frac{x^2+y^2}{2x}$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

(d) Evaluate $\int_3^4 \int_1^2 \frac{1}{(x+y)^2} dy\, dx$.

(e) If $\vec{f} = yzi + zxj + xyk$, then find curl f at $P(1,3,2)$.

(f) If $z = u + v^2$ and $u = e^{x+y^2}$, $v = \sin(y-x)$ then find $\frac{\partial z}{\partial x}$.

(g) State Rolle 's Theorem.

2. (a) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{n!\,3^n}{n^n}$

(b) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}$

3. (a) State Lagrange mean value theorem. Find $'c'$ value of this theorem, if $f(x) = x(x-1)(x-2)$ on $\left[0, \frac{1}{2}\right]$.

(b) Find the evolute of the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

4. (a) Find the maximum value of $f(x,y,z) = x^2y^3z^4$ subject to the condition $2x + 3y + 4z = 18$.

(b) Find the maximum and minimum values of the function $f(x,y) = x^4 + 2x^2y - x^2 + 3y^2$

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5. (a) Evaluate $\int_0^1 \int_{y^2}^{y^{1/3}} (x+y)\,dx\,dy$ by changing the order of integration.

(b) Evaluate $\iint_R (x+y)^2\,dx\,dy$ where R is the region bounded by the parallelogram

$x+y=0, x+y=2, 3x-2y=0,\ 3x-2y=3.$

6. Verify Green's theorem in the plane for $\oint_c (xy+y^2)\,dx + x^2\,dy$ where c is the closed curve of the region bounded by $y=x$ and $y=x^2$.

7. (a) Discuss the convergence of the series $1-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\cdots$.

(b) Find the envelope of the curve $y=mx+am^3$.

Code No: E-5601/N/AICTE

FACULTY OF ENGINEERING

B.E. I Semester (AICTE) (Main & Backlog) (New) Examination, February/ March 2023

Subject: Mathematics- I

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each questions carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. a) Define a sequence with an example.
   b) Find the envelope of a family of curves $y = cx + \frac{1}{c}$, where c is a parameter.
   c) If $x^3 + y^3 - 3xy = 0$, then find $\frac{d^2y}{dx^2}$.
   d) Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dydx.$
   e) Prove that $\nabla(logr) = \frac{\vec{r}}{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$.
   f) Discuss the applicability of Rolle's theorem for $f(x) = tanx$ in $[0, \pi]$.
   g) Show that $\vec{F} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ is solenoidal.

2. a) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$.
   b) Prove that the series $\sum(-1)^n \frac{cosnx}{n\sqrt{n}}$, $x \in \mathbb{R}$ is absolutely convergent.

3. a) State and prove Lagrange's mean value theorem.
   b) Find the centre of circle of curvature of the curve $xy = 1$ at $(1,1)$.

4. a) Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+4y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at the point $(0,0)$.
   b) Using Lagrange's method of multipliers, find the maximum distance of the point $(3,4,12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

5. a) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2} dxdy$ over the annular region enclosed between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ by changing to polar coordinates.
   b) Evaluate $\int_0^1 \int_0^x \int_0^{x+y} e^{x+y+z} dz\, dy dx.$

6. Verify Green's theorem in the plane for $\int_C (2xy - x^2)dx + (x^2 + y^2)dy$, where C encloses the region R bounded by $y = x^2$ and $y^2 = x$.

7. (a) Discuss the convergence of the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots (x > 0)$.

(b) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ in the direction of the vector $-2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ at $(1,2,0)$.

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39

Code No: F-13601/N/AICTE

FACULTY OF ENGINEERING

B.E. I - Semester (AICTE) (Main & Backlog) (New) Examination, February/ March 2024

Subject: Mathematics-I

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each questions carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. a) Discuss the convergence of the series $\frac{1}{\log 2}-\frac{1}{\log 3}+\frac{1}{\log 4}-\frac{1}{\log 5}+\dots$

   b) Using Lagrange's mean value theorem show that $|\cos b-\cos a|\le|b-a|$.

   c) Show that the function $f(x,y)=\begin{cases}\frac{2x^4+3y^4}{x^2+y^2}, & (x,y)\neq 0\\ 0, & (x,y)=0\end{cases}$ is continuous at the point (0,0).

   d) Evaluate the integral $\int_0^1\int_0^x e^{\frac{x}{y}}\,dy\,dx$

   e) If $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A}\times\vec{B}$ is solenoidal.

   f) If $u=x\log xy$ where $x^3+y^3+3xy=1$, find $\frac{du}{dx}$.

   g) Find the directional derivative of $f(x,y,z)=xy^3+yz^3$ at the point (2, -1, 1) in the direction of vector $i+2j+2k$.

2. a) Test for convergence of the series $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n!)}x^{2n}$

   b) Test for convergence of the series $\sum_{n=1}^{\infty}\left[\sqrt[3]{(n^3+1)}-n\right]$

3. a) The function $f(x)=\sin x$ is approximated by Taylor's polynomial of degree three about the point $x=0$. Find c such that the error satisfies $|R_3(x)|\le 0.001$ for all x in the interval [0, c]

   b) Find the coordinates of the centre of curvature at any point of the parabola $y^2=4ax$ and hence find its evolute.

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4. a) If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, then show that $\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2$.

   b) Find the extreme values of $f(x, y, z) = 2x + 3y + z$ such that $x^2 + y^2 = 5$ and $x + z = 1$

5. a) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\, dxdy$ and hence evaluate the same.

   b) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dz\, dx\, dy$.

6. a) Apply Green's theorem to prove that area enclosed by a plane curve C is $\frac{1}{2}\int_C (xdy - ydx)$. Hence, find the area of an ellipse whose semi-major and semi-minor axes are of length a and b.

   b) Verify Stoke's theorem for the vector field $F = (2x - y)i - yz^2 j - y^2 zk$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy - plane.

7. a) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$. Here $r = |\vec{r}|$

   b) If $x = a\cosh\xi\cos\eta$, $y = a\sinh\xi\sin\eta$, show that $\frac{\partial(x, y)}{\partial(\xi, \eta)} = \frac{1}{2}a^2(\cosh 2\xi - \cos 2\eta)$.

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